Size Doesn't Really Matter

Ambiguity Aversion in Ellsberg Urns with Few Balls

Briony D. Pulford and Andrew M. Colman

University of Leicester, UK

Abstract. When attempting to draw a ball of a specified color either from an urn containing 50 red balls and 50 black balls or from an urn containing an unknown ratio of 100 red and black balls, a majority of decision makers prefer the known-risk urn, and this ambiguity aversion effect violates expected utility theory. In an experimental investigation of the effect of urn size on ambiguity aversion, 149 participants showed similar levels of aversion when choosing from urns containing 2, 10, or 100 balls. The occurrence of a substantial and significant ambiguity aversion effect even in the smallest urn suggests that influential theoretical interpretations of ambiguity aversion may need to be reconsidered.

Keywords: ambiguity effect, Ellsberg paradox, payoff variance, urn size effect

Introduction

Ambiguous prospects, with outcome probabilities that cannot be calculated from first principles or estimated from empirical evidence, present a major challenge to decision theory. The issue was first highlighted by Knight (1921), who introduced a distinction between decisions made under *risk* and *uncertainty*, and simultaneously by Keynes (1921), who drew a parallel distinction between *probability* and *weight of evidence*. In Knight's more familiar terminology, a decision is risky when the decision maker does not know what outcome will occur but knows the outcome probabilities, or can judge them with some confidence, and uncertain when the decision maker is ignorant even of the probabilities. Knight illustrated this distinction with an example of two people making blind drawings from an urn containing balls of two colors: "One man knows that there are red and black balls, but is ignorant of the numbers of each; another knows that the numbers are three of the former to one of the latter" (pp. 218–219). The first faces a decision under (unmeasurable) uncertainty, nowadays more commonly called *ambiguity* in psychological literature; the second faces a decision under (measurable) *risk*.

This distinction was ignored or rejected by most subsequent decision theorists, partly because ambiguity is relatively intractable, and partly because decision theorists (e.g., Raiffa, 1961) were quick to point out that we can always apply the *principle of insufficient reason* and assign equal probabilities to the outcomes of an ambiguous choice. Knight (1921) evidently believed that people handle uncertainty in this way: "It must be admitted that practically, if any decision as to conduct is involved, such as a wager, the first man [choosing from the ambiguous urn] would have to act on the supposition that the chances are equal" (p. 219). Savage (1954), in his influential axiomatic subjective expected utility (SEU) theory, brushed ambiguity aside on the grounds that, in order to incorporate it into decision theory, second-order probabilities would be required, and "the introduction of an endless hierarchy seems inescapable. Such a hierarchy seems very difficult to interpret, and it seems to make the theory less realistic, not more" (p. 58). Second-order probabilities can be calculated when outcome probabilities are not known directly but "probabilities of probabilities" can be inferred, as when a decision maker does not know the ratio of red and black balls in an urn but knows that every possible ratio is equally probable.

Ambiguity Aversion

In spite of its theoretical intractability, ambiguity is common in everyday decisions. Furthermore, it is psychologically distinguishable from risk, and empirical evidence contradicts Knight's (1921) assumption that human decision makers merely assign equal probabilities to the outcomes of ambiguous prospects. A substantial body of evidence has shown that most decision makers prefer risky prospects with equal outcome probabilities to ambiguous ones. This is the *ambiguity aversion* effect, and the classic demonstration of it is the Ellsberg paradox (Ellsberg, 1961; Fellner, 1961). In Ellsberg's simplest illustration, two urns are filled with red and black balls, Urn *A* containing 50 red and 50 black balls, randomly mixed, and Urn *B* containing an unknown ratio of 100 red and black balls, randomly mixed. A decision maker chooses either color (red or black) and either urn (*A* or *B*) for a blind drawing and wins a prize if a ball of the chosen color is drawn. Most decision makers

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strictly prefer the known-risk Urn *A* to the ambiguous Urn *B*, irrespective of the preferred color.

Ambiguity aversion violates the axioms of subjective expected utility (SEU) theory, for the following reason. Suppose a decision maker tries to draw a red ball and strictly prefers Urn *A* to Urn *B*. Because the decision maker knows that the probability of drawing a red ball from Urn *A* is 1/2, it can be inferred from SEU theory that the subjective probability of drawing a red ball from Urn *B* must be less than 1/2. It follows that this decision maker's subjective probability of drawing a black ball from Urn *B* must be greater than 1/2, because the two probabilities must sum to unity in the urn, given that the ball must be either red or black. This suggests that the decision maker prefers drawing a black ball from Urn *B* to drawing a red ball from Urn *A*, and the decision to try for a red ball from Urn *A* is therefore inconsistent with the decision maker's own preferences. It fails to maximize SEU and therefore violates SEU theory. Nevertheless, most decision makers prefer the known-risk Urn *A* for both red and black balls, thereby manifesting ambiguity aversion.

Since Ellsberg (1961) discovered this intuitively compelling violation of SEU theory, empirical evidence has confirmed that ambiguity aversion is a powerful and robust phenomenon (Camerer, 1995, pp. 644–649; Camerer & Weber, 1992; Curley & Yates, 1989; Frisch & Baron, 1988; Keren & Gerritsen, 1999; Rode, Cosmides, Hell, & Tooby, 1999). It has been found even when decision makers, without being told the actual ratio of red to black balls in Urn *B*, were told that every possible ratio is equally likely, although this information about second-order probabilities makes the objective chances equal in both urns.

Theoretical Interpretations

Ambiguity aversion is easy to demonstrate but surprisingly hard to explain. It has been suggested (e.g., Krähmer & Stone, 2006; Tetlock, 1991) that it arises from a desire to avoid the anticipated regret that would follow from drawing a losing ball from an ambiguous urn. These and other current theories have no obvious implications regarding the number of balls in the known-risk and ambiguous urns, and the effects of urn size, including very small urns, do not appear to have been systematically investigated. However, there are at least two prominent theories that have strong implications for urn size.

The first is Einhorn and Hogarth's (1985) descriptive model based on the *anchoring and adjustment* heuristic (Slovic & Lichtenstein, 1971). According to this model, a decision maker faced with an ambiguous prospect begins with a provisional probability estimate and then adjusts it up or down on the basis of a mental simulation in which all probability distributions that might apply are imagined, and those that are judged to be inapplicable are excluded. The adjustment is affected by two factors, represented by parameters in the model: the amount of perceived ambiguity (θ), causing a linear increase in the size of the adjustment, and the decision maker's attitude toward ambiguity in the given circumstances (β), causing a further nonlinear adjustment that may vary for different probability values. Einhorn and Hogarth reported four experiments, based on Ellsberg urn choices, in which the model parameters show most decision makers adjusting their subjective probabilities of success in ambiguous urns *down* from 1/2, causing them to prefer known-risk urns.

According to Einhorn and Hogarth (1985), there are special circumstances in which decision makers are likely to prefer ambiguous to known-risk options (pp. 435–436), but "the amount of ambiguity is an increasing function of the number of distributions that are not ruled out (or made implausible) by one's knowledge of the situation" (p. 435). The model does not include any effect of urn size on the initial estimate, although such an effect is quite conceivable. However, the amount of perceived ambiguity θ reflects the "cognitive simulation process" (p. 450), or more specifically "the degree to which one simulates values of *p* that might be" (p. 438), and when a decision maker has relatively sparse relevant information, "one would expect ambiguity to be high because few distributions are ruled out" (p. 442). The model seems to imply greater ambiguity aversion in larger urns, because "there are costs of investing in imagination, increased mental effort and the discomfort that results from greater uncertainty" (p. 459), and above all because the size of the adjustment parameter θ increases monotonically with the number of distributions that need to be imagined and not ruled out (p. 435) – there are more of these distributions in larger urns.

This suggests a clear prediction about urn size. In a mental simulation of an ambiguous urn containing 100 red and black balls in an unknown ratio, there are 101 possible distributions to be imagined, from no red balls to 100 red balls, and none of these is ruled out or excluded. But in a mental simulation of an ambiguous urn containing just two red and black balls in an unknown ratio, there are only three distributions to be imagined and not excluded, namely no red balls, one red ball, and two red balls. In terms of the model, the larger urn is therefore perceived to be vastly more ambiguous than the smaller one. Because only three distributions are imagined and none excluded in the two-ball urn, the model assigns a minute value to the parameter θ that quantifies the amount of perceived ambiguity and determines the size of the ambiguity aversion effect.

A second theoretical approach with strong implications for urn size was put forward by Rode et al. (1999). They suggested that ambiguity aversion arises from decision makers associating ambiguous outcomes with high payoff variability – although our calculations (see Appendix) suggest that variability is not necessarily greater in ambiguous options. According to this approach, ambiguity aversion is a by-product of the application of a risk-sensitive cognitive architecture, adapted by evolution for optimal foraging, that takes account of both the mean and the variance of expected payoffs to minimize the probability that the outcome will fail to satisfy the organism's need (Stephens & Krebs, 1986). If an organism needs *X* calories of food to survive, and if two resource patches have the same mean calorie payoff but different payoff variances, and the mean payoff of the low-variance patch is above *X*, then the organism should forage in that patch; but if the mean payoff in the low-variance patch is below *X*, then it should forage in the high-variance patch. In the extreme case in which the low-variance patch has zero variance (the payoff is certain), the organism is certain to satisfy its need by foraging in that patch if the mean payoff is above the threshold *X* and certain to fail if the mean is below *X*; but if the mean is below *X*, then foraging in the high-variance patch yields a small but positive probability of satisfying its need. Hence, according to Rode et al., decision makers "are not avoiding ambiguity per se: instead, they are avoiding the high variance of outcomes of ambiguous options" (p. 296).

According to Rode et al. (1999), ambiguity aversion arises from an overgeneralization of this policy to decisions in which one of the options has an unspecified distribution. They provided evidence to support the conjecture that human decision makers tend to avoid ambiguous options only when known-risk options meet their needs. First, they showed that the size of the ambiguity-aversion effect tends to increase with the probability of success in the known-risk option. Second, in an experiment in which decision makers had to choose between a known-risk option with obviously *high* payoff variability and an ambiguous option with obviously *low* payoff variability, most chose the low-variability option. These results reversed the standard ambiguity aversion effect and provided further corroboration for this interpretation.

In ambiguous urns in which every possible distribution is equally likely, as they were in our experiment and those of Rode et al. (1999), it is possible to calculate expected payoff variances exactly (see Appendix). The expected variance in an ambiguous urn containing *n* balls, calculated as the mean of the *n* equiprobable variances that might apply to the urn, turns out to be $1/6 - 1/6n$. Hence, the expected payoff variance increases rapidly for small values of *n* and never exceeds 1/6. The expected payoff variance is very small for a 2-ball ambiguous urn (0.08) and much larger in 10-ball and 100-ball ambiguous urns (0.15 and 0.16, respectively). These expected payoff variances are smaller, not larger, than the variances for known-risk urns. In a known-risk urn of any size with 50% balls of each color, the payoff variance is 0.25. This result seem to be at odds with the theory of Rode et al. (1999), which interprets ambiguity aversion as a consequence of variance avoidance.

Urn size clearly has relevance to the interpretation of ambiguity aversion. If the effect turns out to be unaffected by urn size, and particularly if ambiguity aversion is found even in very small urns, then an explanation of it will have to include something in addition to the cognitive mechanisms suggested by Einhorn and Hogarth (1985) and Rode et al. (1999). Evidence from other areas of research suggests that decision makers are sometimes sensitive to urn sizes. For example, Denes-Raj and Epstein (1994) showed that many people preferred drawing from a large urn than from a smaller one with fewer winning balls but a larger *proportion* of winning balls, even when they understood that the probability of winning was greater in the small urn. Typically, they preferred to draw from an urn containing seven winning balls among 100, rather than from an urn containing one winning ball among 10. Introspective reports suggested that they preferred the larger urn because it offered more ways of winning.

To clarify the possible effects of urn size on ambiguity aversion, we therefore investigated choices in a standard Ellsberg urns task, using urns of widely different sizes, from the conventional 100 balls down to just two balls.

Method

Participants

The sample consisted of 151 undergraduate students and members of the general public (100 women and 51 men) with a mean age of 23.03 years (*SD* = 10.24, range 16 to 76). Prizes of £30 sterling were awarded to three lottery winners, with entry to the lottery being dependent on drawing a blue ball from an urn containing red and blue balls. The responses of two participants were illegible and were discarded, reducing the usable sample size to 149.

Materials

Three pairs of urns were used, each urn containing red and blue balls. The pairs differed according to the number of balls in each urn: 2, 10, or 100 balls. For each urn size, one of the urns contained 50% red and 50% blue balls (the known-risk urn), and the other contained a randomly selected ratio of red and blue balls (the ambiguous urn).

Design and Procedure

This experiment was designed to examine the effects of urn size (2, 10, or 100 balls) on urn choice (known-risk or ambiguous urn) using an independent-groups experimental design. The ratio of red to blue balls was known to the decision makers in the known-risk urn and was unknown in the ambiguous urn. Participants were told that if they picked a blue ball, they would be entered into a lottery with the chance of winning one of three £30 prizes. They were free to choose from either the known-risk or the ambiguous urn. Participants were randomly assigned to these three treatment conditions, and they began by filling in consent forms and providing demographic and contact details. Those assigned to the 100-ball condition were then presented with the following written instructions:

Consider the following problem carefully, then write down your decision. On the table are two urns, labeled *A* and *B*, containing red and blue marbles, and you have to draw a marble from one of the urns without looking. If you get a blue marble, you will be entered into a £30 lottery draw.

Urn *A* contains 50 red marbles and 50 blue marbles. Urn *B* contains 100 marbles in an unknown color ratio, from 100 red marbles and 0 blue marbles to 0 red marbles and 100 blue marbles. The mixture of red and blue marbles in Urn *B* has been decided by writing the numbers 0, 1, 2, . . ., 100 on separate slips of paper, shuffling the slips thoroughly, and then drawing one of them at random. The number chosen was used to determine the number of blue marbles to be put into Urn *B*, but you do not know the number. Every possible mixture of red and blue marbles in Urn *B* is equally likely.

You have to decide whether you prefer to draw a marble at random from Urn *A* or Urn *B*. What you hope is to draw a blue marble and be entered for the £30 lottery draw. Consider very carefully from which urn you prefer to draw the marble, then write down your decision below. You will draw a marble from your chosen urn straight afterwards.

I prefer to draw a marble from Urn A/Urn B . . . .

Minor alterations were made for the treatment conditions with smaller urns, replacing the number 100 with either 2 or 10. So, for example, the two-ball condition read "the mixture of red and blue marbles in Urn *B* has been decided by writing the numbers 0, 1, 2 on separate slips of paper, shuffling the slips thoroughly, and then drawing one of them at random" and the 10-ball condition read "the mixture of red and blue marbles in Urn *B* has been decided by writing the numbers 0, 1, 2, . . ., 10, on separate slips of paper, shuffling the slips thoroughly, and then drawing one of them at random."

Each participant drew a ball from the chosen urn, and those who drew blue balls were entered into the prize lottery. The ratios of red to blue balls in the ambiguous urns were decided randomly, as described in the written instructions. In the two-ball condition, the known-risk urn contained one red and one blue ball, and in the ambiguous urn the randomization procedure resulted in two red balls. In the 10-ball condition, the known-risk urn contained five red and five blue balls, and the ambiguous urn eight red and two blue balls. In the 100-ball condition, the known-risk urn contained 50 red and 50 blue balls, and the ambiguous urn 53 red and 47 blue balls.

Results

Of the 149 decision makers who participated in the experiment, 106 (71%) chose the known-risk urn, and 43 (29%) chose the ambiguous urn. This finding replicates the basic ambiguity aversion effect across urn sizes, $\chi^2(1, N = 149) = 26.64$, $p < .001$, effect size $w = .42$ (medium). In this experiment, the same number of men and women chose the ambiguous urns.

Results for different urn sizes are shown in Table 1.

Table 1. Choices of known-risk and ambiguous urns of three different sizes

	Urn chosen	
Urn size	Known risk	Ambiguous
2	29 (64.44%)	16 (35.56%)
10	36 (78.26%)	10 (21.74%)
100	41 (70.69%)	17 (29.31%)
Total	106 (71.14%)	43 (28.86%)

Strong ambiguity aversion effects occurred in all urn sizes, and urn choice was not significantly influenced by urn size: $\chi^2(2, N = 149) = 2.12$, $p = .35$, *ns*. To provide a more severe test of the effect of urn size, data from urn sizes of 10 and 100 were collapsed to determine whether urn choice differed significantly between smallest (2) and larger sizes (10 or 100), but the association remained nonsignificant: $\chi^2(1, N = 149) = 1.41$, $p = .24$, *ns*. Furthermore, if urn sizes 2 and 10 are collapsed and compared with urn size 100, the association is still nonsignificant: $\chi^2(1, N = 149) = 0.92$, $p = .53$, *ns*. Taken together, these results provide clear-cut confirmation of the finding that the number of balls, and hence the number of possible permutations of colors in the ambiguous urn, had no significant effect on urn choice and hence ambiguity aversion.

Discussion

Only 29% of the decision makers chose ambiguous urns, replicating the fundamental ambiguity aversion effect across urn sizes. The participants knew that every possible distribution of balls in the ambiguous urns was equally probable, with the obvious implication that the objective chances were equal in the known-risk and ambiguous urns, but a medium-sized ambiguity aversion effect occurred nonetheless. This is hardly a new finding, but the occurrence of a substantial and significant ambiguity aversion effect even in the smallest urn fails to confirm predictions implied by two leading theoretical interpretations of ambiguity aversion (Einhorn & Hogarth, 1985; Rode et al., 1999).

According to Einhorn and Hogarth's (1985) model, smaller urns should necessarily be perceived as less ambiguous than larger ones, because far fewer distributions need to be imagined and excluded in the mental simulation that is hypothesized to occur during the process of judgment and decision making, and according to the model's equations, this should reduce the size of the ambiguity aversion effect. We found no urn size effect, and a significant ambiguity aversion effect occurred even in the smallest urn, containing just two balls. With only three distributions to simulate, namely no red balls, one red ball, and two red balls, compared to 101 distributions in the largest 100-ball urn, and none to exclude in either case, the ambiguity aversion effect should have been eliminated or at least greatly attenuated, yet it remained significant even in this very small urn. The risk-sensitive foraging theory of Rode et al. (1999) also predicts very little ambiguity aversion in the smallest urn, because expected payoff variance is close to zero when there are only two balls in the urn, and our findings are therefore inconsistent with that theory also. If ambiguity aversion is related to variance avoidance, then our findings suggest that decision makers have a tolerance for payoff variance up to some threshold above $\sigma^2 = 0.25$ and that, for all urn sizes, they tend to prefer known-risk alternatives for some other reason.

Whatever accounts for ambiguity aversion, our finding that the effect remained significant in the smallest urn seems difficult to reconcile with the purely cognitive theories that we have considered in this article. In spite of the vastly smaller number of distributions in the smallest urn, careful data analysis failed to reveal evidence of any diminution of ambiguity aversion. One possibility is that ambiguity aversion is driven by the range of probabilities of success – the range was from zero to unity in urns of all three sizes in our experiment – rather than the number of distributions that need to be mentally simulated (as suggested by Einhorn & Hogarth, 1985) or the expected payoff variance (as suggested by Rode et al., 1999). What is most revealing is the positive finding of a significant effect in the smallest urn, and this needs to be taken into account in any interpretation of ambiguity aversion.

We have provided preliminary rather than conclusive evidence that we hope will inspire further research into urn size effects. Our experimental design was restricted to between-subjects urn size comparisons, to avoid confounding urn size with subject-expectancy effects, although a within-subjects design might possibly have made urn size more salient and caused decision makers to have been more sensitive to these differences (cf. Denes-Raj & Epstein, 1994). If decision makers were presented in a future study with choices between ambiguous urns of different sizes, then a significant preference for smaller urns would provide evidence in favor of theories of ambiguity aversion, such as those of Einhorn and Hogarth (1985) and Rode et al. (1999), that imply different degrees of aversion in urns of different sizes. On the other hand, an absence of any significant urn size preferences would be consistent with theories that have no obvious implications for urn size, including the interpretation that we suggest below. However, irrespective of any between-subjects or within-subjects urn size comparisons, our finding of significant ambiguity aversion in the smallest urn is inconsistent with purely cognitive theories that imply that ambiguity aversion arises from the effort involved in mentally simulating the possible distributions or avoiding high-variance options. Our research was also restricted to comparing preferences for 50–50 known-risk urns with ambiguous urns containing unknown numbers of winning balls between 0 and 100 per cent, although we acknowledge that preferences for restricted-range ambiguous urns with (for example) between 40 and 60 per cent winning balls also deserve investigation.

With these caveats in mind, we believe that existing theories of ambiguity aversion may need to be reconsidered in the light of our findings. Our results show that ambiguity aversion occurs when decision makers are unable to quantify the risks involved in ambiguous options, even when the outcome sets are easily cognitively simulated and the expected payoff variance is very small. We agree with Rode et al.'s (1999) finding that ambiguity aversion is caused by aversion to the unknown probability parameter and is not due to a comparative process. We suggest that ambiguity aversion may arise from a more general intolerance of uncertainty, and in particular from the aversive and disturbing effects of uncertainty, irrespective of urn size. Most people prefer to avoid exposing themselves to events and circumstances that they do not understand (Becker & Brownson, 1964; Freeston, Rhéaume, Letarte, Dugas, & Ladouceur, 1994; Furnham, 1994; Ghosh & Ray, 1997), and ambiguity aversion may be a particular manifestation of this. Uncertainty induces a disturbing and aversive psychological state. However, there are large individual differences in intolerance of uncertainty. Habitual worriers tolerate uncertainty less well than others and appear to be especially prone to define ambiguous prospects as threatening (Butler & Mathews, 1983, 1987), and this may explain why some people display more ambiguity aversion than others. Purely cognitive interpretations that ignore such affective processes are unlikely to provide a complete explanation of ambiguity aversion.

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Briony D. Pulford

School of Psychology
University of Leicester
Leicester LE1 7RH
UK
E-mail bdp5@le.ac.uk

Appendix

Expected Payoff Variance in Ambiguous Urns

An ambiguous urn contains n balls, of which k ($k = 0, 1, \ldots, n$) are red and the rest black, with every value of k equally likely. A decision maker draws a ball and receives a payoff of $x = 1$ if it is red.

The probability of a red ball is 1/2, by symmetry. Formally,

$$\frac{1}{n+1}\sum_{k=0}^{n}\frac{k}{n} = \frac{1}{n(n+1)}\sum_{k=0}^{n}k = \frac{1}{n(n+1)}\left[\frac{1}{2}n(n+1)\right] = \frac{1}{2}.$$

In an urn containing exactly k red balls, the expected payoff $E(x) = \mu = k/n$. By definition, the variance $\sigma^2 = E(x-\mu)^2 = E(x^2 - 2\mu x + \mu^2)$. Because μ is a constant, $\sigma^2 = E(x^2) - 2\mu E(x) + \mu^2$, and because $\mu = E(x)$,

$$\sigma^2 = E(x^2) - 2[E(x)]^2 + [E(x)]^2 = E(x^2) - [E(x)]^2 = \frac{k}{n}1^2 - \frac{k^2}{n^2} = \frac{k}{n} - \frac{k^2}{n^2}.$$

We first prove by induction that $\sum_{k=0}^{n} k^2 = [n(n+1)(2n+1)]/6$.

For $n = 0$, the formula reduces to $0 = 0$, which is true. We then prove that if it holds for n, then it must also hold for $n+1$.

$$\sum_{k=0}^{n+1} k^2 = \sum_{k=0}^{n} k^2 + (n+1)^2.$$

Using the expression for $\sum_{k=0}^{n} k^2$ assumed above, this is equal to $[n(n + 1)(2n + 1)]/6 + (n + 1)^2 = [(n + 1)/6][n(2n + 1) + 6(n + 1)]$, which simplifies to $[(n + 1)/6][2n^2 + 7n + 6] = [(n + 1)/6][(n + 2)(2n + 3)] = [(n + 1)/6](n + 2)[(2n + 1) + 1]$, and this is equal to $\{[(n + 1)][(n + 1) + 1][2(n + 1) + 1]\}/6$, as required. Therefore, for all n,

$$\sum_{k=0}^{n} k^2 = [n(n+1)(2n+1)]/6.$$

The sum of variances for all values of k is

$$\sum_{k=0}^{n}\left(\frac{k}{n}-\frac{k^2}{n^2}\right)=\sum_{k=0}^{n}\frac{k}{n}-\sum_{k=0}^{n}\frac{k^2}{n^2}.$$

This is equal to

$$\frac{1}{n}\sum_{k=0}^{n} k-\frac{1}{n^2}\sum_{k=0}^{n} k^2=\frac{1}{n}\frac{n(n+1)}{2}-\frac{1}{n^2}\frac{n(n+1)(2n+1)}{6},$$

which simplifies to $(n + 1)/2 - [1/n][(n + 1)(2n + 1)]/6$. The expected variance is thus

$$E(\sigma^2)=\frac{1}{n+1}\left[\frac{(n+1}{2}-\frac{1}{n}\frac{(n+1)(2n+1)}{6}\right].$$

Therefore, $E(\sigma^2) = 1/2 - (2n + 1)/6n = 1/2 - 1/3 - 1/6n = 1/6 - 1/6n$. This expression measures the expected payoff variance in an ambiguous urn.

The expected variance tends to 1/6 as $n \to \infty$. For $n = 2$, $E(\sigma^2) = 1/6 - 1/12 = 1/12 \approx 0.083$; for $n = 10$, $E(\sigma^2) = 9/60 = 0.150$; and for $n = 100$, $E(\sigma^2) = 99/600 = 0.165$.